Packet Success Probability Derivation in a Vehicular Ad Hoc Network for a Highway Scenario

Ali Rakhshan
and Hossein Pishro-Nik
School of Electrical and
Computer Engineering
University of Massachusetts
Amherst, Massachusetts
Emails: {arakhshan, pishro}@ecs.umass.edu

***Abstract*—This paper studies the probability of packets being successfully received by vehicles (packet success probability) for chains of vehicles on a highway by taking multi-user interference, path loss, and fading into account. Our simulation results compare two scenarios, the case in which vehicles follow Poisson distribution and the case when the distance between any two adjacent vehicles is equal. This probability plays a main role in vehicles' collision probability; therefore, this mathematical framework to obtain packet success probabilities can be employed in designing the safety applications of vehicular ad hoc networks, especially the newly proposed customized vehicular communications.**

I. INTRODUCTION[1]

During the past decade, the automobile industry has seen a rise in the use of advanced technologies, such as state-of-the-art electronic devices, in order to improve automobile safety. Sadly, however, the fatalities and injuries caused due to automobile accidents have remained at an alarming level. In particular, statistics from 2013 [1] report over five million crashes in the U.S., causing over two million injuries and more than 30,000 fatalities.

A major cause of accidents is the slow response time of drivers to stopped traffic, i.e., the average time a driver takes to hit the brake after a preceding car has stopped. The cumulative response times for the leading vehicles play the main role in the collision probability [2] of the upstream vehicles, potentially resulting in domino-style collisions. To reduce the drivers' response time to accidents, recent research and development in the automobile industry has introduced collision warning systems to be installed on modern automobiles. Collision warning systems are capable of cautioning about critical, time-sensitive incidents such as crashes or traffic jams.

With the advancements in Vehicular Ad Hoc Networks (VANET)(Fig. 1) , recent research [2] suggests the use of VANETs to improve the effectiveness of collision warning systems. VANETs allow for cross-communication between cars within a close proximity of each other, which can enable them to efficiently and reliably communicate sensitive traffic messages such as crash-relevant information. The Federal Communications Commission has allocated 75 MHz of spectrum in the 5.9 GHz band for Dedicated Short Range Communications (DSRC). To serve as the groundwork for DSRC, the IEEE 802.11p standard was published in the year 2010 for Wireless Access in Vehicular Environments (WAVE) [3].

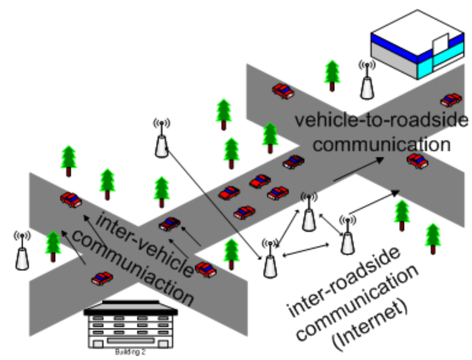

Fig. 1. VANET: Vehicular Ad-hoc NETwork

The 75 MHz spectrum of DSRC is divided into seven 10 MHz-wide channels. One channel is called the control channel (CCH) and serves exclusively for safety messages. The other channels are called service channels (SCH) and are reserved for commercial applications. Safety messages are either event-driven or periodic. Each vehicle sends periodic messages in a single hop regularly in order to inform other vehicles inside its given neighborhood of important information such as location, speed, and acceleration while it sends event-driven messages to warn other vehicles of a collision.

In order to ameliorate drivers' safety, first we need to know the delay requirements of the safety applications. In general, the difference between the sum of perception reaction times of drivers in a chain and communication delay plays the main role in reducing the average collision probability of the vehicles. Perception reaction time (PRT) is the time needed for a driver to perceive that something has happened and react to it.

We need to know about the uncertainty of the packet delivery between two specific vehicles while other vehicles

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[2]Hereafter, the term collision shall refer to vehicle collisions unless explicitly stated to denote packet collisions.

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might also transmit simultaneously, thus interfering with the selected packet transmission. Deriving this probability helps us with finding the communication delay to inform each vehicle in a chain while employing vehicular communications. It is desirable to reduce this delay as much as possible by lessening the interference caused by other vehicles.

Our main contributions in this paper are as follows:

1) We find the expression of packet success probability for two specific scenarios regarding a chain of vehicles on a highway.
2) We illustrate the collision probability for the specified models using simulations.

The remainder of this paper is organized as follows. Section II summarizes the related work that has been done in the field of vehicular communications regarding finding the communication parameters suitable for the delay requirements of safety applications. We propose our MAC level design to obtain packet success probability equations in section III. In section IV, the simulation results are demonstrated. Conclusion is given in section V.

II. Background and Literature Review

[4] simulated two vehicular safety applications and determined the effect of various communication parameters on vehicle crash avoidance through simulations. However, they don't develop any mathematical framework for packet delivery success. [5] proposes a series of repetition-based Media Access Control (MAC) protocols to deliver periodic status updates within their useful lifetime to within a specified range. For a scheme in which nodes transmit with a given probability in each slot, [5] derives the Probability of Reception Failure (PRF) at the border of the range of interest. However, they only consider the strongest interferer in their derivation and neglect fading. The authors do not mention how their design meets the specific packet reception probabilities and delay requirements. [6] develops a stochastic model in which they derive the average number of collisions (when the leading vehicle stops instantly) in a chain of vehicles that are equipped with a collision warning system. The operation of the communications system is abstracted by a message delay variable whose distribution is assumed given for any specific MAC scheme. However, it is assumed that all vehicles in the chain receive the warning message at the same time. This assumption is not realistic because the communication delay depends directly on the packet success probability of the vehicle to vehicle communication. [7] compares the safety of automated and manual highway systems with respect to rear-end collision frequency and severity. Yet, they assume a fixed communications delay for autonomous, low-cooperative, and high-cooperative vehicles, respectively, an assumption which we show may not be sensible.

III. Analysis and Design

Communications between vehicles can help drivers with making proper reactions to the deceleration events especially when a driver cannot either observe or perceive the deceleration of other vehicles due to low visibility, high unexpectedness of the incident, defected brake lights, and many distractions that nowadays exist on the roads. In a network of vehicles, each vehicle transmits with a specific probability in the transmission medium. Large channel access probabilities lead the system to excessive interferences and consequently low probability of packets being successfully received (success probability) while very small values reduces the success probabilities since the probability of the favorite transmission is low itself.

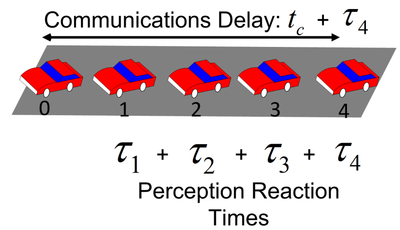

Fig. 2. Communications delay versus sum of PRTs. This figure illustrates the time before a driver in a chain applies the brake.

A. Delay Requirements of the Safety Application

Consider a traffic stream where a chain of vehicles move with constant speed v and randomly chosen inter-vehicle spacing. When V_0 (the first vehicle in the chain) brakes, the driver of V_1 (the following vehicle), after her PRT, τ_1, applies the brake. Having no inter-vehicle communications employed, vehicle $V_i \quad (i > 1)$ applies the brake after $\sum_{j=1}^{i} \tau_j$, the sum of PRTs up to the driver i. With the communications, this time will change to $\tau_i + t_c$ in which t_c is the communications delay to inform vehicle V_i. Note that t_c can be a result of direct communications from V_0 to V_i or the retransmission of V_0's signal by one of the vehicles in the middle. Understandably, when $t_c < \sum_{j=1}^{i-1} \tau_j$, which is almost always the case, V_i has more time to react and as a result the collision probability is reduced (Fig. 2).

B. Two Scenarios

[8] states that vehicles traffic are more likely to follow Poisson distribution under low flow conditions. Under near-capacity conditions, however, the equal distance assumption between vehicles is justified. Therefore, our design is divided into two cases: 1. Equal distance model 2. Poisson distribution model. We believe examining these two scenarios gives us a thorough picture of how vehicular communications can affect collision probability in general.

Although the Media Access Control (MAC) protocol for DSRC communications is a variation of the conventional CSMA/CA scheme, because of the short length of the packet payload and the broadcast nature of communications, the 4-way handshake anticipated by the standard is not efficient for the dissemination of periodic safety messages.

RTS/CTS and ACK message exchanges increase the hidden node problem thus resulting in higher probability of packet collisions [10]. Since the topology of VANETs is highly dynamic, we need protocols which do not need a detailed description of the network topology to schedule packet transmissions. Repetition-based protocols not only reveal this property, but also fight packet collisions due to the problem of hidden nodes. Hence, in this section, we make use of repetition-based protocols for the dissemination of periodic safety messages. A similar approach has been used in other papers, e.g. in [11] and [10].

1) Equal distance:
The MAC scheme that we consider is SSP (Slotted Synchronous P-persistent) where at each slot a node (vehicle) transmits with probability p and receives with probability $1-p$ independent of others. The important assumption is that the slots are synchronized because of the on-board GPS devices. Moreover, since the vehicles are not faced with power constraints, the nodes can increase the transmission power to overcome the interference. In this paper, we consider path loss and Rayleigh fading for formalizing the signal propagation characteristics. If we assume that the nodes transmit with unit power, the received power at distance r is $hr^{-\alpha}$, where $\alpha(>1)$ is the path loss exponent and h is the fading coefficient. Assuming that a node transmits a packet, the probability that a receiver at distance r receives the packet successfully is:

$$\begin{aligned} P_s &= P\left(\frac{S}{I} > \beta\right) \\ &= P\left(\frac{hr^{-\alpha}}{\sum_{i=-\infty}^{\infty} b_i h_i r_i^{-\alpha}} > \beta\right) \\ &= \frac{\lambda(1+\beta)}{p_{tr} + \lambda(1-p_{tr}) + \lambda(1-p_{tr})\beta} \cdot \\ &\prod_{i=-\infty-\{0\}}^{+\infty} \frac{p_i + \lambda(1-p_i) + \lambda(1-p_i)\beta\left(\frac{m}{i}\right)^{\alpha}}{\lambda\left(1+\beta\left(\frac{m}{i}\right)^{\alpha}\right)} \end{aligned} \quad (1)$$

Proof: see Appendix A.

where β is the SIR decoding threshold, b_i is a Bernoulli random variable with parameter p_i, node i transmits with probability p_i (the specified transmitter transmits with probability p_{tr}), r_i denotes the distance from the interferer i to the receiver (Fig. 3), λ is the fading exponential parameter, and i and m denote the index of interferer i and receiver respectively. Our assumption is that vehicles are located around the receiver to infinity symmetrically. If the channel access probabilities are equal, the closed-form packet success probability is ($\alpha = 2$):

$$P_s = \frac{(1+\beta)[p+\lambda(1-p)]^2}{\lambda(1-p)[p+\lambda(1-p)+\lambda(1-p)\beta]} \cdot \frac{\left[\sinh \pi \sqrt{\frac{\lambda(1-p)\beta}{p+\lambda(1-p)}}m\right]^2}{\left(\sinh\sqrt{\beta}m\right)^2} \quad (2)$$

Proof: see Appendix B.

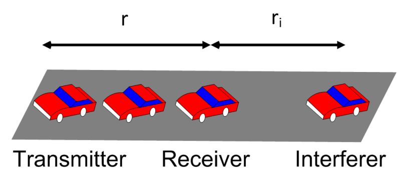

Fig. 3. A chain of vehicles. Distance between the transmitter and the desired receiver $= r$. Distance between interferer i and desired receiver $= r_i$.

If x denotes the distance between two adjacent nodes, mx represents the distance between receiver and transmitter. It is noteworthy to mention that equations 1 and 2 do not depend on the inter-vehicle distance.
There are two approaches for an N-lane highway. The first approach is called the Single Lane Abstraction (SLA) model. In this model, all the traffic lanes are mapped into one lane with the aggregated traffic intensity. Using this model, equations 1 and 2 can still be employed to obtain packet success probability. SLA model can be used only if $d^2 \ll mx^2$ in which d shows the distance between two adjacent lanes (see Appendix C). If this condition is not satisfied, we cannot ignore d. Therefore, packet success probability can be obtained using:

$$P_s = \frac{\lambda(1+\beta)}{p_{tr} + \lambda(1-p_{tr}) + \lambda(1-p_{tr})\beta} \cdot \prod_{i\in-\infty-\{0\}}^{+\infty} \frac{p_i + \lambda(1-p_i) + \lambda(1-p_i)\beta\left(\frac{mx}{ix+\frac{d^2}{2ix}}\right)^2}{\lambda\left(1+\beta\left(\frac{mx}{ix+\frac{d^2}{2ix}}\right)^2\right)} \quad (3)$$

Proof: see Appendix D.
If the time slots in which nodes transmit are not synchronized, this scheme is named *Slotted Asynchronous P-persistent* (SAP). In this case, an interferer can potentially interfere with *at most* two time slots of another transmission. Hence, the transmission probability for the interferers is:

$$p_i' = p_i + p_i - p_i \cdot p_i \simeq 2p_i \quad (4)$$

Since the probabilities are small, this approximation is tight.

2) Poisson Distribution:
Poisson point processes have been widely employed as a model for wireless networks [17]–[19]. In this case, the nodes are distributed on a highway according to a Poisson point process. The packet success probability can be obtained by considering the fact that the transmitter-receiver distance is a random variable, not a constant value.

$$
\begin{aligned}
P_S &= P(SIR > \beta) && (5)\\
&= \int_r P\left(\frac{Phr^{-\alpha}}{k+I} > \beta\right) f_R(r)dr && (6)\\
&= \int_r P\left(h > \frac{\beta(k+I)r^{\alpha}}{P}\right) f_R(r)dr && (7)\\
&= \int_r e^{\frac{-\lambda\beta k r^{\alpha}}{P}} \cdot E_I\left[e^{\left(\frac{\lambda\beta r^{\alpha}}{P}\right)}\right] f_R(r)dr && (8)\\
&= \int_r e^{\frac{-\lambda\beta k r^{\alpha}}{P}} \cdot L_I\left(\frac{\lambda\beta r^{\alpha}}{P}\right) f_R(r)dr && (9)\\
&\text{if} \quad (k=0, \alpha=4, P=1)\\
&= \int_r L_I\left(\lambda\beta r^4\right) f_R(r)dr && (10)
\end{aligned}
$$

where P, h, r, α, k, I, and λ represent the transmitter signal power, the channel fading, the distance between transmitter and receiver, the path loss exponent, the noise variance, the interference, and the exponential parameter of Rayleigh fading respectively. Assuming the transmitter and receiver are located on the same lane, the distribution of the distance between transmitter and receiver is Erlang.

$$
f_R(r) = \frac{\lambda_p^n r^{n-1} e^{-\lambda_p r}}{(n-1)!} \tag{11}
$$

in which λ_p represents the intensity of vehicles on a lane. Also, n denotes the number of nodes between transmitter and receiver plus one. [9] obtains closed-form expressions for the Laplace transform of the approximate aggregate interference. For this specific scenario, this Laplace transform is equal to:

$$
\begin{aligned}
&L_I\left(\lambda_p\beta r^4\right) = \\
&e^{-\pi\lambda_M\left[b^2(1-e^{-\lambda_p\beta K(\frac{r}{b})^4})+(\lambda_p\beta r^4 K)^{\frac{1}{2}}\Gamma(0.5,0.5Kb^{-4})\right]}
\end{aligned}
\tag{12}
$$

in which

$$
\Gamma(s,x) = \int_x^{\infty} t^{s-1} \cdot e^{-t} dt \tag{13}
$$

$$
K = \left(\frac{c}{4\pi f_c}\right)^2 \tag{14}
$$

Also, b, c, f_c represent the radius from the receiver node in which the aggregate interference is considered, the speed of radio propagation, and the carrier frequency. λ_M denotes the intensity of the simultaneously active nodes from the parent Poisson point process.

$$
\lambda_M = \frac{1-e^{-\lambda_p L}}{\lambda_p} \tag{15}
$$

Proof: see Appendix E (L denotes the length of a lane). To summarize, in order to obtain this intensity, the concept of marked point processes is employed [16]. It is often useful to include additional information about the points in the model. Thus, in marked point processes each point x_i is assigned a random variable, the mark m_i. It is necessary to choose m_i in a smart way in order to model the spatial distribution of the active set of interferers.

IV. Numerical and Simulation Evaluation of Design

Communications delay is a main factor that influences the collision probability. Also, we know that some of the vehicles are too far from the vehicle V_0 (the leading vehicle) to be able to receive the messages directly from it. Thus, when one of the vehicles in the middle gets informed and reacts to the event, the message will be forwarded to the vehicles at a greater distance from the leading vehicle. In other words, after a vehicle in the middle starts decelerating, the new status will be included in the new messages from this vehicle to further upstream vehicles. Therefore, we need to compute the time it takes for a message to be received by vehicle i. It is sufficient that the message be received successfully only one time. As a result the successful reception at vehicle V_i has a geometric distribution with parameter

$$
P_s(i) \cdot p_{tr} \cdot (1-p_i) \tag{16}
$$

where $P_s(i)$ is a packet success probability, e.g. given in equation 1. Also, p_{tr} and p_i represent the channel access probability for the transmitter and the desired receiver respectively. This parameter demonstrates the probability that the transmitter is sending messages, the desired receiver is obtaining the warnings, and the warning messages are successfully delivered, all simultaneously. This gives us the number of required slots on average for vehicle V_i to receive vehicle V_0's messages:

$$
s(i) = \frac{1}{P_s(i) \cdot p_{tr} \cdot (1-p_i)} \tag{17}
$$

If SAP scheme is employed, we need to alter the equation:

$$
s(i) = \frac{1}{P_s'(i) \cdot p_{tr} \cdot (1-p_i')} \tag{18}
$$

in which p_i' represents the channel access probability when the time slots are not synchronized and $P_s'(i)$ denotes packet success probability using the new channel access probabilities.

The allowable number of transmission opportunities within the tolerable delay period is:

$$
D = \lfloor \frac{T(i)R}{L} \rfloor \tag{19}
$$

TABLE I
IEEE 802.11P DATA RATES AND CORRESPONDING SIR DECODING THRESHOLDS

R (Mbps)	3	4.5	6	9	12	18	24
β (db)	5	6	8	11	15	20	25

TABLE II
SIMULATION PARAMETERS

Distribution	Poisson Equal distance
Velocity	$20\frac{m}{s}$
Deceleration rate	$[-4,-8]\frac{m}{s^2}$
Number of vehicles in an specific lane	8
Total number of vehicles	30
SIR decoding threshold	11 dB
Data rate	9 Mbps
Packet length	250 Bytes

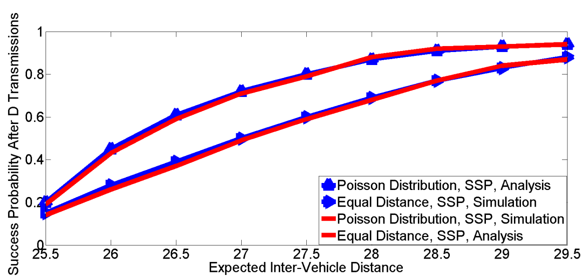

Fig. 4. Packet success probability after D transmissions at vehicle V_2 for different traffic models and different expected inter-vehicle distance.

R represents the data rate which is chosen from TABLE I while L denotes the packet length. $T(i)$ denotes the maximum tolerable delay to inform vehicle V_i which can be obtained from TABLE I. Let P_s^D denote the success probability at V_j after D transmission opportunities:

$$P_s^D = 1-(1-s(j)^{-1})^D$$

This equation demonstrates the dependence of packet success probability on channel access probabilities and inter-vehicle distances (Fig. 4).

Fig. 5 illustrates the advantage of employing the customized communications [12], assuming a different number of vehicles are placed on that part. If we use the same simulation parameters for the equal-distance scenario, even greater reduction in collision probabilities are achieved. This seems to be justifiable because the equal-distance model represents the dense traffic, thus more collisions happen. The two extreme scenarios which we considered in this paper lead us to the conclusion that we may achieve a model-free approach to improve the performance of VANETs since both of the examined scenarios benefit the drivers following a similar pattern.

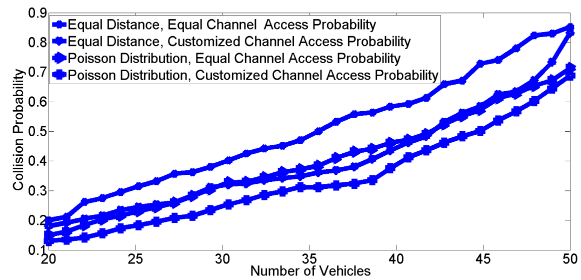

Fig. 5. Collision probability versus the number of vehicle. The other parameters are chosen from Table II. The comparison is between four cases (Vehicle locations, Communications): 1. Equal distance, equal channel access probability. 2. Equal distance, customized channel access probability. 3. Poisson distribution, equal channel access probability. 4. Poisson distribution, customized channel access probability. Customized channel access probability means two constant values 0.07 and 0.03 are assigned to safe and unsafe vehicles respectively. Safe vehicles are the ones with higher distances to their preceding vehicles ($>$ 15 meters) and drivers with lower perception reaction times ($<$ 1s). Also, the equal channel access probability and equal distance values are assumed to be 0.05 and 30 meters respectively. The mean of Poisson distribution is also 30 meters.

V. CONCLUSION

In this paper, we derived the equations of packet success probability for two traffic models in a network of vehicles. By taking these equations into account in the design of collision warning systems, the fatalities on highways will be reduced.

APPENDIX A
PROOF OF EQUATION 1

If there is distance r between a transmitter and the desired receiver, the success probability is

$$P_s = P\left(\frac{hr^{-\alpha}}{I} > \beta\right) \tag{20}$$

$$= \int P(h > \beta r^{\alpha} I | I = i) f_I(i) di \tag{21}$$

$$= E\left[e^{-\beta r^{\alpha}\sum_{i\in\Phi} b_i h_i r_i^{-\alpha}}\right] \tag{22}$$

$$= \prod_{i\in\Phi}\left[\frac{p_i}{\lambda\left(1+\beta r^{\alpha} r_i^{-\alpha}\right)} + (1-p_i)\right] \tag{23}$$

Assuming,

$$r = mx \quad \text{and} \quad r_i = ix \tag{24}$$

equation 1 is obtained.

APPENDIX B
PROOF OF EQUATION 2

Using Euler's product formula, we obtain the second equation.

$$sin(\pi z) \equiv \pi z \prod_{i=1}^{\infty}\left(1-\frac{z^2}{i^2}\right) \tag{25}$$

APPENDIX C
DIFFERENCE BETWEEN SLA MODEL AND MULTI-LANE MODEL

Assume d is the distance between two specific lanes, x denotes the distance between two adjacent vehicles, and the transmitter is located on the middle lane. Let's assume r specifies the distance between transmitter and receiver (which is on a lane with distance d from the middle lane).

$$r = mx\sqrt{1+\left(\frac{d}{mx}\right)^2} \tag{26}$$

$$\approx mx\left(1+\frac{\left(\frac{d}{mx}\right)^2}{2}\right) \tag{27}$$

$$= mx + \frac{d^2}{2mx} \tag{28}$$

Therefore, if $d^2 \ll mx^2$, $r \approx mx$. Hence, we can assume the receiver is in the middle lane too. If not, that approximation cannot characterize the performance of vehicular networks on N-lane highways.

APPENDIX D
PROOF OF EQUATION 3

In appendix A, the last equation is modified with respect to the new assumption that the inter-lane distance cannot be overlooked:

$$P_s = \prod_{i\in\Phi}\left[\frac{p_i}{\lambda\left(1+\beta\left(\frac{mx}{ix+\frac{d^2}{2ix}}\right)^{\alpha}\right)} + (1-p_i)\right]$$

Then, equation 3 will be obtained.

APPENDIX E
PROOF OF EQUATION 15

Let's assume $\Phi = \{n_i; i = 1, 2, 3, \cdots\}$ are the nodes in the network. The probability of having n points in a lane (lane length = L) is

$$P(|\Phi| = n) = \frac{(\lambda L)^n e^{-\lambda L}}{n!}$$

A marked point (n_i, m_i) is selected to be retained if and only if it has the lowest mark m_i in a circle of radius r centered at n_i. r is the guaranteed exclusion distance between any two simultaneously active transmitters. This model is called HCPP [16] which is built by applying dependent thinning to the parent point process Φ. If we assume that the distribution of the marks in one circle is Uniform, then the probability of retaining a random point can be written as:

$$P = \sum_{n=0}^{\infty}\frac{1}{n+1}P(\text{having n points in the lane}) \tag{29}$$

$$= \frac{1-e^{-\lambda_p L}}{\lambda_p L} \tag{30}$$

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